Osherson, Smith and Elkan

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Orange Is The New Apple

Rosch

“Psychology and Concepts”

- In a series of papers Eleanor Rosch gives conclusive evidence that human concepts do not have sharp boundaries
- Eventually Rosch surmises that Fuzzy Set Theory might be useful to model concepts

Osherson and Smith

"On the adequacy of prototype theory as a theory of concepts"

- 1981 $\rightarrow$ Influential psychologists Osherson and Smith (O&S) list the problems with fuzzy logic as a basis for concept theory
- 1982 $\rightarrow$ A second O&S paper virtually ends the investigation into fuzzy set theory by Psychology

Belohlavek and Klir & Wierman

Criticizing O&S

- 2011 $\rightarrow$ Belohlavek and Klir critique the Mathematics of O&S
  - A book. The journals were not interested.
- 2013 $\rightarrow$ Wierman critiques the Philosophy of O&S
  - All but one of O&S's arguments date back to ancient Greece.

Induction

Fact

It is well known, for example, that the principle of mathematical induction cannot be reliably applied to, vague predicates.

- Define F to be the numerical predicate:
  ____ grains of sand brought together do not constitute a heap.
  - 0 grains of sand brought together do not constitute a heap
  - if k grains won't do the job neither will $k+l$
- So mathematical induction leads to the false conclusion that no matter how large k gets, k grains of sand brought together do not constitute a heap.

Heap

Piles of Sand

- Zeno (c. 490 – c. 430 BC) used this argument to prove
  - infinity does not exist.
- O&S used this argument to prove
  - vagueness does not exist.
- Max Black notes that if you do pile up enough sand,
  - you do get a heap,
  - mathematical induction is not applicable here,
  - emergence!

Proof

Axiom (Peano)

AB=BA

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Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Then $AB = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $BA = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$. Since this violates my axiom, which is assumed to be true, we can conclude that those mathematicians who have defined matrix multiplication are all mixed up! □

Concept Models

Currently, there are three major views of concept (Machery 2009) :

- Prototypes
- Exemplars
- Theory.

Exemplar

The Set Theory–Propsitional Logic view

- Concepts are represented in the mind by a group of exemplars.
- The mind stores the best examples of birds and matches other objects to these exemplars.

Prototypes

The Predicate Logic view

- Concepts are represented in the mind by prototypes that are built over time.
- As we see more birds we come to understand what makes a bird birdy.
- The closer, in terms of a multi–attribute distance, an object is to the prototype, the greater its identification with the concept.

Theory

The Object Oriented View

- Theory theorists posit that concepts are like scientific theories.
- Thus a bird is not just attributes, but activities, interactions, and behaviors.
- The mind needs concepts for explanation as well as recognition.

Kamp and Partee

- Unfortunately for O&S Fuzzy Set Theory is surely the right tool to model vagueness.
- Kamp and Partee (K&P) in 1995, while preferring supervaluation, gave fuzzy set theory a lot of lip service.
- In 1997 O&S strike back with a new paper “On typicality and vagueness”.

Typicality

Prototype?

$$c^{p}_{con}$$

- a function that maps the domain of discourse into real numbers,
  - in practice only positives,
  - like *utility* only order matters.

Vagueness

Exemplar?

$$c^{e}_{con}(o)$$

- "graded applicability of a concept"
- They allow that these values may be confined to the unit interval $[0, 1]$.
- No examples!

Chairs

- Obviuos Truths
- The Queen of England's Throne *Q* is a chair but it is a poor example of a chair.
- An IKEA "chair" box, when assembled provides a vary good example *I* of a chair.

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- A Striped–Apple is a fair example of an Apple.
- A Striped–Apple is a fair example of Striped.
- If a Striped–Apple is a conjunction using **min** then a Striped–Apple is only a fair example of a Striped–Apple.

Bully

- Consider two 13-year-olds, Butch and Marvin.
  - Butch beats up 9- year-olds for fun,
  - pushes 8-year-olds off their bicycles, and is
  - the terror of the 7- year-olds.
- He wears jeans and a leather jacket, and has perfect eyesight.
- Marvin is just like Butch except that he
  - wears tortoise shell glasses to correct a mild astigmatism.
- Most people would agree that both are bullies.

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- It follows that $c^e_{bully} \neq c^t_{bully}$ contradicting (4a) with respect to bully.

Zadeh

A note on prototype theory and fuzzy sets.

" ... an object may be far from the prototype in terms of a given metric and yet have full membership in A".

- A Candy Apple is a fine Apple to Zadeh.

Elkan

Fact

"While a proposition θ may be uncertain without further clarification, we must expect that $\theta \vee \neg\theta$ will be true. Its negation $\theta \wedge \neg\theta$ is also certainly false."

Elkan

It is easy to prove that if, for arbitrary propositions θ and ϕ , we have that

$$\theta \wedge \neg\theta = \phi \wedge \neg\phi$$

then the range of propositions must be restricted to $\{0, 1\}$.

Problems

The Psychology of O&S

Many pathological problems

- Universe
- Zero and One
- Absolutes

The Universe

O&S logic and mathematical formulas

- Assume $c^e_{apple}(o)$ is a function
- What is the universe?
  - The set of all objects leads to Russell's Paradox,
  - The set of all apples is a circular definition,
  - O&S tap dance around this problem.

Vagueness

O&S never use any values but zero or one.

- Not once is a striped apple given an actual vague grade of appleness.
  - An apple is, but how do we know what an apple is the first time we see it?.

Truth

O&S are desperate for an absolute truth

- An apple is surely an apple, $A = A$,
- The apple that is not an apple,
- The law, of the excluded middle,
- Obviously the chair from an IKEA box labeled “chair”.

O&S tell psychologists to abandon fuzzy set theory.

- Really they are clinging to absolutism,
- Psychologists had largely abandoned prototypes.

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Strange Logic

- It happens $1/3$ of the time is probability and acceptable
- It is true $1/3$ is vague and unacceptable,
- Scientists, philosophers and mathematicians love
  - **truth**
  - it is so nice and clean
  - so absolute
  - but don't ask me to show you an object that is truth!
- There are going to be no absolutes in a proper theory of concepts.

Psychology

Web of Science

Psychology is the (non-engineering) field having the most works citing Zadeh's seminal paper.

- learning disability,
- developmental progression,
- psychotherapy,
- interests and occupational preferences,
- fitting in organizational psychology,
- false memories,
- user's web navigation patterns,
- analysis of questionnaires,
- developmental psychology,
- work adjustment to retirement transition,
- signal detection analysis,
- linguistic prototypes.