Irrational Choice

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Outline

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Rational Model

- The Rational Model is a relational model with two main assumptions.
  - The first is that given a pair of options, *a* and *b*, the actor must decide between
    - "*a* as good as *b*,"
    - "*b* as good as *a*," or
    - both "*a* as good as *b* and *b* as good as *a*."
  - The second assumption is one of transitivity
- X is a set of social alternatives with $|X| \geq 3$.
- $N = \{1, \ldots, n\}$ with $n \geq 2$ is a finite set of individuals (actors, judges, etc.).

Outline

Relational Systems

relation: a subset of $X \times X$.
We may use $x\,r\,y$ and $r(x,y) = 1$ to *denote* that $(x,y) \in r$.

reflexive: $r(x,x) = 1$.

complete: $r(x,y) = 1$ or $r(y,x) = 1$.

symmetric: $r(x,y) = r(y,x)$.

anti–symmetric: when $x \neq y$, we have that $r(x,y) \neq r(y,x)$.

transitive: if $r(x,y) = 1$ and $r(y,z) = 1$ then $r(x,z) = 1$.

Crisp Preference

Definition (Preference)

A relation r is is a preference relation if it is complete and transitive.

- Crisp preference r can be split into two components,

  Indifference $i(x,y) = 1$ iff $r(x,y) = r(y,x) = 1$.
  Strict Preference $p(x,y) = 1$ iff $r(x,y) = 1$ and $r(y,x) = 0$.

- Then $r = i \cup p$ and
  - indifference is symmetric
  - strict preference is anti–symmetric.

Subtle Assumptions

- These preferences are stable and fixed.
- Actors
  - Have no *memory*
  - Have no *strategy*
  - Never *learn*
  - Never *interact*
  - No *organization*
  - Never interaction with the *environment*

Important Results?

- The most famous results of rational choice are those of Arrow [1] and Sen [2].
- Under a *reasonable* set of assumptions about social preference, Arrow shows that dictatorship is inevitable.
- Sen shows *Individual Vetos* produce sub–optimal Social Choice.
- There are *many* critiques about the *reasonableness* of the assumptions of these proofs
  - Sen's [3] own critique of the "rational fool."

Outline

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   - Crisp Relations
   - Fuzzy Rational Model
2. Problems with Fuzzy Relation Properties
   - The Epsilon Problem
   - The crisp problem
   - What is indifference?
   - The Utility Problem
3. An Irrational Choice
   - Example
   - Assessment
4. Conclusions

Arrow Obsession

- Banarjee [4] was one of the first to examine whether or not Arrow's results follow when we use a fuzzy relation R instead of a crisp relation r.
- Of course, this study mandates that the concepts of rational choice embodied in crisp set theory be extended into fuzzy set theory.

Definition (Fuzzy relation)

R is a fuzzy subset of $X \times X$, so $R(x,y) \in [0,1]$.

Fuzzy Relations

Everyone Agrees That..

reflexive $R(x,x) = 1$.

symmetric $R(x,y) = R(y,x)$.

anti–symmetric when $x \neq y$, we have that $R(x,y) \neq R(y,x)$.

Fuzzy Relations?

Not Everyone Agrees That..

complete $\max\{R(x,y),R(y,x)\} > 0$.

strongly connected $\max\{R(x,y), R(y,x)\} = 1$.

connected $R(x,y)+R(y,x) \geq 1$.

transitive if $R(x,y) = \alpha$ and $R(y,z) = \beta$ then $R(x,z) \geq \min[\alpha,\beta]$.

Outline

The Epsilon Problem

Definition (Epsilon)

$$\varepsilon = 1.0 \times 10^{-13} \quad (1)$$

- If $R(a,b)$ differs from $R(b,a)$ by epsilon R is not symmetric.
- Will $temperature_1 = temperature_2$ after
  - calibration,
  - measurement,
  - encoding,
  - transmission, and
  - processing?
- For computers $a = b$ if $Math.abs(a-b) \leq \varepsilon$

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The Crisp Problem

Naive fuzzification produces non-equivalent fuzzy set definitions.

- A crisp concept can have many equivalent definitions.
- An crispy condition for completeness is that

$$r(a,b)+r(b,a)-r(a,b)r(b,a)>0$$

- If $R\in[0,1]$ then

$$R(a,b)+R(b,a)-R(a,b)R(b,a)>0$$

  is not the same as

$$\max\{R(x,y),R(y,x)\}>0$$

- Another crispy condition for completeness is that $r(a,b)+r(b,a)\geq 1$ ...

Outline

Indifference

A foolish consistency is the hobgoblin of little minds–Emerson

- Direct translation of crisp into fuzzy.

Decomposition

The symmetric (I) and anti–symmetric (P) components of R

$$I(a,b) = I(b,a) = \min\left[R(a,b), R(b,a)\right] \ ,$$

$$P(a,b) = \begin{cases} R(a,b) & R(a,b) > R(b,a) \\ 0 & \text{otherwise} \end{cases} .$$

- Since $I \cup P = R$ everything is great, right?

Indifference and Meaning

"In a fuzzy world, is symmetry the characteristic of indifference?"

- This definition avoids thinking about what the numbers mean.
- If R is symmetric to begin with then $I = R$ and we have total indifference.
- However we now have many different levels of indifference.
- How can this be made operational?

Indifference

I will put much more effort into abstaining?

Likert scales

An Essential tool of the social sciences.

Question:

"I found the conference interesting..."

- SD Strongly disagree
- D Somewhat disagree
- N Undecided
- A Somewhat agree
- SA Strongly agree

Neutrality and Indifference

- A Likert scale gives ordinal valued data.
- All that we know is that $SD < D < N < A < SA$.
- If we wish to take this Likert data and apply fuzzy set methods we need to map the answers to membership grades, see Wierman and Tastle [5].
- Typically $\langle SD, D, N, A, SA \rangle$ maps to $\left\langle 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1 \right\rangle$.
- When we use a Likert scale neutrality is obviously $1/2$.
- But neutrality must not be analogous to indifference?

Outline

Utility

- Implicitly, the choice of one option over another is based on some subjective utility function u which maps options to non–negative values such that $r(a,b) = 1$ if and only if $u(a) \geq u(b)$.
  - Utility is unbounded?
  - Greed is good!
- Unfortunately utility *must* be subjective:
  - winning a million dollars would alter your lifestyle quite a bit;
  - if Bill Gates won a million dollars he would just give it away.

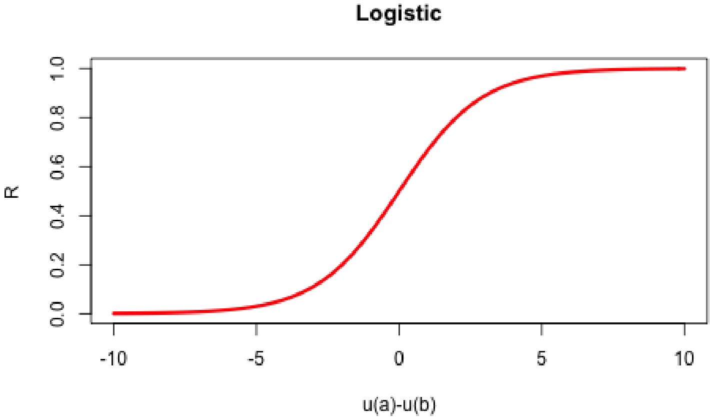
Utility to Choice

There must be some functional methodology leading from utility to preference.

- Suppose that
$$R(a,b) = \rho(u(a), u(b)) \, .$$
- Since $R(a,b) = \rho(u(a), u(b))$ and $R(b,a) = \rho(u(b), u(a))$ any reasonable assumptions about ρ, such as *continuity* and *monotonicity*, would impose regularity conditions on R that are not assumed in most of the literature.
- The independence of $R(a,b)$ and $R(b,a)$ directly conflict with methods such as the Analytic Hierarchy Process (AHP).
    - AHP automatically sets the value of $Q(b,a)$ to $1/Q(a,b)$.

Logistic functions

- If we were to use utility to create a fuzzy preference relation then an obvious first choice is the logistic function.



Logistic Properties

- The logistic formula (with steepness $k > 0$)

$$R(a,b) = \frac{1}{1 + e^{-k(u(a)-u(b))}}\,.$$

- The logistic function has many desirable properties
  - it is very sensitive when $u(a)$ and $u(b)$ are close together, and
  - very insensitive when $u(a)$ and $u(b)$ are far apart.
  - When $u(a)$ and $u(b)$ are equal $R(a,b) = R(b,a) = 0.5$
    - this is a much better model of indifference.
  - it is complementary
    - $R(a,b) + R(b,a) = 1$.

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Example

Irrationality

R	a	b	c
a	1	0.49	0
b	0	1	0.51
c	0	0	1

- The relation R is not transitive
  - $R(a,b) = 0.49$ and $R(b,c) = 0.51$ require that $R(a,c)$ be, at least, 0.49.
- The relation R is not complete since $\mathrm{R(a,c)} = R(c,a) = 0$ and rational choice says we must decide.
  - Set $\mathrm{R(a,c)} = \varepsilon$!
- Strangely, rational choice has no problem with $\mathrm{R(a,c)} = R(c,a) = 1$.

Outline

A Different Approach

Maybe we should measure reflexivity, transitivity, etc.

Reflexivity Measures Average the diagonal:

$$ref(R) = \frac{\sum_x R(x,x)}{n}.$$

Symmetry Measures Average the distance from $R(a,b)$ to $R(b,a)$. There are $K = \frac{(n-1)(n-2)}{2}$ connections so

$$sym(R) = 1 - \frac{\sum_{1 \le i < j \le n} \left| R(x_i,x_j) - R(x_j,x_i) \right|}{K}.$$

Assessment

Transitivity Measures Let R° be the transitive closure of R and use subsethood

$$tran(R) = Subsethood(R, R^\circ).$$

Completeness Measures Completeness makes no sense in a fuzzy relation. Change $0 \mapsto \varepsilon$ and its complete.

Connectedness The average of how often $R(a,b)$ to $R(b,a)$ sum to $1+$.

$$con(R) = \frac{\sum_{x,y \in X} \min\{1, R(x,y) + R(y,x)\}}{n^2}.$$

What the paper shows

Crunchy Thinking

The fuzzification of rational choice is a lively area of research.

Fuzzy Thinking

Perhaps an entirely different approach is needed.

Implication

Fuzzy is Hard!

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