Psychologists: Are They Logically Fuzzy?

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Outline

1. Concepts
   - History
   - Logic

2. Osherson and Smith
   - Four Problems

3. O&S Logic
   - Concepts in Psychology
   - Absolutism

4. Conclusions
   - Truth and Red
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Serendipity

“Concepts: Are They Logically Fuzzy?”

- NAFIPS 2010 → George Klîr tells me that he is publishing a book about fuzzy logic and concepts
- NAFIPS 2012 ⇒ Eleanor Rosch gives a talk: “Concepts: Are They Logically Fuzzy?”
- Summer 2012 ⇒ I buy the book edited by Belohlavek and Klîr and find Rosch wrote chapter 3
Osherson and Smith

"On the adequacy of prototype theory as a theory of concepts"

- 1981 $\rightarrow$ Osherson and Smith (influential psychologists) list the problems with fuzzy logic as a basis for concept theory
- 1982 $\Rightarrow$ Osherson and Smith’s second paper ends the investigation into fuzzy set theory by Psychology
  - Google “Osherson and Smith”
- 2011 $\Rightarrow$ Belohlavek and Klir critique the mathematics of O&$S$
  - A book. The journals were not interested.
Paradox

1976 – The new math teacher proves that you can’t prove everything
1977 – I become obsessed with Logic and read Frege, Russell, Kleene, Łukasiewicz, Gödel · · ·
1978 – My Senior Thesis is about Paradox
1980 – I take logic and set theory in Grad School
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Paradoxes and Fallacies

O&S From a Logical Perspective

1. Begging the Question
2. The Problem of Future Contingents
3. Induction
4. Conjunction
Begging the Question

Prohibition

- Recognized by Aristotle in his *Prior Analytics*.
- Basically the conclusion is inherent in the assumption.
  - “It must be wrong or else it would not be illegal.”
The Problem of Future Contingents

- Aristotle, Chapter 9 of *On Interpretation*.
  
  Suppose a Sea Battle will not be fought tomorrow. If this is true now, then it was also true yesterday. In fact it must have always been true. A thing that is always true is simply true. Therefore a Sea Battle can not be fought tomorrow.
  
  Aristotle concludes that the law of the excluded middle is NOT valid for future events.
Define $F$ to be the numerical predicate:

___ grains of sand brought together do not constitute a heap.

- 0 grains of sand brought together do not constitute a heap
- if $k$ grains won’t do the job neither will $k + 1$

So mathematical induction leads to the false conclusion that no matter how large $k$ gets, $k$ grains of sand brought together do not constitute a heap.

Zeno used this argument to prove infinity does not exist.
Conjunction

Example by Tversky and Kahneman

*Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.*

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Even though answer two is more specific than answer one, 90% of those asked choose number 2.
Conjunction

- The measure of the conjunction of two events is always less than or equal to the measure of either one occurring alone.
- This is true for probability and any measure, even a fuzzy measure.
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Let $SA$, $S$ and $A$ be the fuzzy sets for Striped–Apple, Striped and Apple respectively.

It cannot be that $SA(x) > S(x) \land A(x)$

Abandon fuzzy sets.

O&S (and Psychology) should also abandon probability!
2 O&S Empty

"Apple that is not an Apple"

- Must be the empty set!
- The fuzzy set $A \land \neg A$ is not empty!
- Begging the question!

*If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.*
3 O&S Disjunction

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<thead>
<tr>
<th>Person</th>
<th>Liquidity</th>
<th>Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$105,000</td>
<td>$5,000</td>
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<tr>
<td>B</td>
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Fuzzy can not solve this problem

Liquidity, $L(A) > L(B) > L(C)$
Investment, $I(A) < I(B) < I(C)$
and for Wealth, “Intuitively”,
$W(B) > W(A)$ and
$W(B) > W(C)$
### Liquidity, Investment & Wealth

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and for Wealth, “Intuitively”,
$W(B) > W(A)$ and $W(B) > W(C)$

Thank god!
“All grizzly bears are native to North America”

- The birth of a grizzly bear in a Martian Zoo contradicts this statement.
- O&S cast this as a formula to make it seem more logical:

\[ \forall x \ GB(x) \rightarrow NA(x) \]

- O&S abandon fuzzy set theory
- If O&S abandon logic then they can not write their paper.
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Classically, in psychology a concept was associated with a set.

- In this view either an object $x$ was a *bird* or it was not a *bird*.
- Rosch performed a series of experiments that challenged this simple view of category.
- Her work indicated that, in the human mind, a concept was graded.
- Objects matched concepts more or less, and sooner or later.
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The O&S Problem

The arguments of O&S should lead us to abandon

- logic,
- rationality,
- relations,
- probability,
- measure, and, of course,
- fuzzy set theory.

O&S tell psychologists to abandon fuzzy set theory.

- Really they are clinging to absolutism
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Conclusions

Truth and Red

Fuzzy Logic

All of the examples of O&S are

- Historically old problems with logic, or
- Assumption of absolutes that beg the question,
- The stuff philosophy majors make a living on.
- Accepted as truth in Psychology
What color is red?

- Why is this such a hard question?
- Everyone knows what red is don’t they?
- Rosch gave overwhelming evidence that the psychological concept red was imprecise.
  - But, of course, overwhelming evidence does not make it true.
Apples

Premise
This apple is very red.

Inference rule
If a apple is red then the apple is ripe.

Deduction
The apple is very ripe.

Humans want the machines to pick the ripe apples.