

Psychologists: Are They Logically Fuzzy?

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Outline

- 1 Concepts
 - History
 - Logic
- 2 Osherson and Smith
 - Four Problems
- 3 O&S Logic
 - Concepts in Psychology
 - Absolutism
- 4 Conclusions
 - Truth and Red



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Serendipity

“Concepts: Are They Logically Fuzzy?”

- NAFIPS 2010 → George Klir tells me that he is publishing a *book* about fuzzy logic and concepts
- NAFIPS 2012 ⇒ Eleanor Rosch gives a talk: “Concepts: Are They Logically Fuzzy?”
- Summer 2012 ⇒ I buy the *book* edited by Belohlavek and Klir and find Rosch wrote chapter 3



Osherson and Smith

“On the adequacy of prototype theory as a theory of concepts”

- 1981 → Osherson and Smith (influential psychologists) list the problems with fuzzy logic as a basis for concept theory
- 1982 ⇒ Osherson and Smith’s second paper ends the investigation into fuzzy set theory by Psychology
 - Google “Osherson and Smith”
- 2011 ⇒ Belohlavek and Klir critique the mathematics of O&S
 - A book. The journals were not interested.



Paradox



- 1976 – The new math teacher proves that you can't prove everything
- 1977 – I become obsessed with Logic and read Frege, Russell, Kleene, Łukasiewicz, Gödel ...
- 1978 – My Senior Thesis is about Paradox
- 1980 – I take logic and set theory in Grad School



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Paradoxes and Fallacies

O&S From a Logical Perspective

- 1 Begging the Question
- 2 The Problem of Future Contingents
- 3 Induction
- 4 Conjunction



Begging the Question



Prohibition

- Recognized by Aristotle in his *Prior Analytics*.
- Basically the conclusion is inherent in the assumption.
 - “It must be wrong or else it would not be illegal.”



The Problem of Future Contingents

- Aristotle, Chapter 9 of *On Interpretation*.
 - Suppose a Sea Battle will not be fought tomorrow. If this is true now, then it was also true yesterday. In fact it must have always been true. A thing that is always true is simply true. Therefore a Sea Battle can not be fought tomorrow.
- Aristotle concludes that the law of the excluded middle is NOT valid for future events.



Induction

Osherson and Smith's second paper

- Define F to be the numerical predicate:

_____ grains of sand brought together do not constitute a heap.

- *0 grains of sand brought together do not constitute a heap*
- *if k grains won't do the job neither will $k + 1$*
- So mathematical induction leads to the false conclusion that no matter how large k gets, k grains of sand brought together do not constitute a heap.
- Zeno used this argument to prove infinity does not exist.



Conjunction

- Example by Tversky and Kahneman

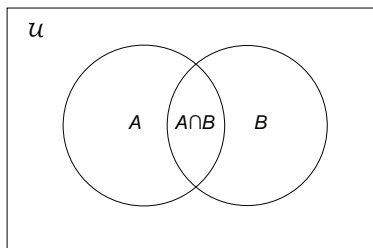
Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

- Which is more probable?
 - 1 Linda is a bank teller.
 - 2 Linda is a bank teller and is active in the feminist movement.
- Even though answer two is more specific than answer one, 90% of those asked choose number 2.



Conjunction

- The measure of the conjunction of two events is always less than or equal to the measure of either one occurring alone.
- This is true for probability and any measure, even a fuzzy measure.



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1 O&S Conjunction



Apples

- Let SA , S and A be the fuzzy sets for Striped–Apple, Striped and Apple respectively.
- It cannot be that $SA(x) > S(x) \wedge A(x)$
- Abandon fuzzy sets.
- O&S (and Psychology) should also abandon probability!



2 O&S Empty



"Apple that is not an Apple"

- Must be the empty set!
- The fuzzy set $A \wedge \neg A$ is not empty!
- Begging the question!

*If it looks like a duck, swims like a duck,
and quacks like a duck, then it probably
is a duck.*



3 O&S Disjunction

Person	Liquidity	Investment
A	\$105,000	\$5,000
B	\$100,000	\$100,000
C	\$5,000	\$105,000

Fuzzy can not solve this problem

Liquidity, $L(A) > L(B) > L(C)$

Investment, $I(A) < I(B) < I(C)$

and for Wealth, "Intuitively",

$W(B) > W(A)$ and

$W(B) > W(C)$



Liquidity, Investment & Wealth

Person	Liquidity	Investment
A	\$105,000	\$5,000
B	\$10,000	\$10,000
C	\$5,000	\$105,000

Fuzzy can not solve this problem

Liquidity, $L(A) > L(B) > L(C)$

Investment, $I(A) < I(B) < I(C)$

and for Wealth, “*Intuitively*”,

$W(B) > W(A)$ and

$W(B) > W(C)$

- Thank god!

4 O&S Truth

“All grizzly bears are native to North America”

- The birth of a grizzly bear in a *Martian* Zoo contradicts this statement.
- O&S cast this as a formula to make it seem more logical:

$$\forall x GB(x) \rightarrow NA(x)$$

- O&S abandon fuzzy set theory
- If O&S abandon logic then they can not write their paper.



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A Bird

Classically, in psychology a concept was associated with a set.

- In this view either an object x was a *bird* or it was not a *bird*.
- Rosch performed a series of experiments that challenged this simple view of category.
- Her work indicated that, in the human mind, a concept was graded.
- Objects matched concepts more or less, and sooner or later.



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The O&S Problem

The arguments of O&S should lead us to abandon

- logic,
- rationality,
- relations,
- probability,
- measure, and, of course,
- fuzzy set theory.

O&S tell psychologists to abandon fuzzy set theory.

- Really they are clinging to absolutism



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Fuzzy Logic

All of the examples of O&S are

- Historically old problems with logic, or
- Assumption of absolutes that beg the question,
- The stuff philosophy majors make a living on.
- Accepted as truth in Psychology



Red

What color is red?

- Why is this such a hard question?
- Everyone knows what red is don't they?
- Rosch gave *overwhelming evidence* that the psychological concept red was imprecise.
 - But, of course, overwhelming evidence does not make it true.



Apples

Premise

This apple is very **red**.

Inference rule

If an apple is **red** then the apple is ripe.

Deduction

The apple is very **ripe**.

Humans want the machines to pick the **ripe** apples.

